### Consumer surplus (CS)

<table>
<thead>
<tr>
<th>Demand: ( P = f(Q) = 18 - \frac{1}{2}Q )</th>
<th>Supply: ( P = g(Q) = 3 + \frac{1}{3}Q )</th>
</tr>
</thead>
</table>
| \[
    \text{CS} = \frac{(18 - 9) \cdot 18}{2} = 81
    \] | \[
    \text{PS} = P^eQ^e - A - B
    = 9 \cdot 18 - \frac{(9 - 3) \cdot 18}{2} - 3 \cdot 18 = 54
    \text{or } \text{PS} = \frac{(9 - 3) \cdot 18}{2} = 54
    \] |
| Formula: \[
    \text{CS} = \int_0^{Q^e} (18 - \frac{1}{2}Q) \, dQ - P^eQ^e
    \] | Formula: \[
    \text{PS} = \int_0^{Q^e} (3 + \frac{1}{3}Q) \, dQ
    \] |
| \[
    \text{CS again} = \int_0^{Q^e} (18 - \frac{1}{2}Q) \, dQ - P^eQ^e
    = 18Q - \frac{1}{4}Q^2 - P^eQ^e
    = 18 \cdot 18 - \frac{1}{4} \cdot 18^2 - 9 \cdot 18 = 81
    \] | \[
    \text{PS again} = \int_0^{Q^e} (3 + \frac{1}{3}Q) \, dQ
    = P^eQ^e - 3Q - \frac{1}{6}Q^2
    = 9 \cdot 18 - 3 \cdot 18 - \frac{1}{6} \cdot 18^2 = 54
    \] |

### 7.1 Consumer surplus (CS)

Demand: \( P = 15 - Q \) \( (P^e = 9) \)

Calculate consumer surplus (diagram and formula).
### Consumer surplus (CS)

**Demand:** 
\[ P = 32 - Q - \frac{1}{10}Q^2 \quad (Q^e = 10) \]

Calculate consumer surplus (formula).

### Producer surplus (PS)

**Supply:** 
\[ P = 5 + 2Q \quad (Q^e = 4) \]

Calculate producer surplus (diagram and formula).

### Producer surplus (PS)

**Supply:** 
\[ P = \frac{1}{2}Q^2 + Q + 2 \quad (P^e = 26) \]

First calculate \( Q^e \) (quadratic equation), then calculate producer surplus (formula).

### Consumer surplus (CS) and producer surplus (PS)

**Demand:** 
\[ P = 32 - 8Q_d \]

**Supply:** 
\[ P = 12 + 2Q_s \]

Calculate consumer surplus and producer surplus at the market equilibrium.

### Consumer surplus (CS) (Monopoly vs competition)

**Demand (monopoly):** 
\[ P = AR = 30 - 2Q \]

Marginal cost (MC) of the monopolist = 12

#### 7.61 Calculate Q and P if the monopolist targets a maximum profit.

#### 7.62 Calculate consumer surplus: Monopoly vs competition

<table>
<thead>
<tr>
<th>Monopoly</th>
<th>P &gt; MC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Competition</td>
<td>P = MC</td>
</tr>
</tbody>
</table>

[We assume that in the case of competition demand and marginal cost are the same as in the case of monopoly.]

#### 7.63 Graph 7.61 and 7.62 in the same diagram and verify both CS.

### Consumer surplus (CS), producer surplus (PS) and the effects of a per unit tax

**Demand:** 
\[ P = 240 - 6Q_d \]

**Supply:** 
\[ P = 120 + 4Q_s \]

Answering 7.71 and 7.72 graph two different diagrams.

#### 7.71 Calculate consumer surplus and producer surplus at the market equilibrium.

#### 7.72 Now a new per unit tax of 20 is introduced. Calculate the tax receipts and the new CS, the new PS as well as the deadweight loss.
7 Consumer surplus and producer surplus

### 7.1 Consumer surplus (CS)

**Graph:**
- Demand curve (P vs. Q)
- Consumer surplus (CS) area

**Formula:**
\[ Q^e: \quad 9 = 15 - Q^e \rightarrow Q^e = 6 \]

**Calculation:**
\[ CS = \frac{1}{2} (15 - 9) \times 6 = 18 \]

**Result:**
\[ CS = 18 \]

### 7.2 Consumer surplus (CS)

**Formula:**
\[ P^e = 32 - 10 - \frac{1}{10}10^2 = 12 \]

**Calculation:**
\[ CS = \int_0^6 (32 - Q - \frac{1}{10}Q^2) \, dQ = 32Q - \frac{1}{2}Q^2 - \frac{1}{30}Q^3 \quad P^eQ^e = 15 \times 6 - \frac{1}{2}6^2 - 6\times 9 = 18 \]

**Result:**
\[ CS = 1162 \]

### 7.3 Producer surplus (PS)

**Graph:**
- Supply curve (P vs. Q)
- Producer surplus (PS) area

**Formula:**
\[ P^e = 5 + 2\times 4 = 13 \]

**Result:**
\[ PS = 13 \]
7.3 cont.

\[ PS = P^e Q^e - A - B = 13*4 - \frac{4*8}{2} - 4*5 = 52 - 16 - 20 = 16 \]

\[ PS = P^e Q^e - \int_0^4 (5 + 2Q) \, dQ = 13*4 - 5Q - Q^2 = 52 - 5*4 - 4^2 = 52 - 20 - 16 = 16 \]

7.4 Producer surplus (PS)

\[ Q^e \]
- \[ 26 = 0.5Q^2 + Q + 2 \]
  - \[ 0.5Q^2 - Q + 24 = 0 \]
  - \[ Q^2 + 2Q - 48 = 0 \]
- Factorization:
  \[(Q + 8)(Q - 6) = 0\]
  \[Q_1 = -8 < 0 \rightarrow (Q \text{ must be positive.})\]
  \[Q_2 = 6\]
  \[Q^e = 6\]
- Formula:
  \[\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 + 4*48}}{2} = \frac{-2 + 14}{2} = 6 \text{ and } \frac{-2 - 14}{2} = -8\]
  \[Q^e = 6\]

\[ PS = P^e Q^e - \int_0^6 (\frac{1}{2}Q^2 + Q + 2) \, dQ = 26*6 - \frac{1}{6}Q^3 - \frac{1}{2}Q^2 - 2Q = 156 - \frac{1}{6}6^3 - \frac{1}{2}6^2 - 2*6\]
  \[= 156 - 36 - 18 - 12 = 90\]

7.5 Consumer surplus (CS) and producer surplus (PS)

- Market equilibrium:
  \[Q^e: \quad 32 - 8Q^e = 12 + 2Q^e \]
  \[10Q^e = 20\]
  \[Q^e = 2\]
  \[P^e = 32 - 8*2 = 16\]
- \[CS = \int_0^2 (32 - 8Q) \, dQ = -P^e Q^e = 32Q - 4Q^2 - 16*2 = 32*2 - 4*2^2 - 32\]
  \[= 64 - 16 - 16 = 16\]
- \[PS = P^e Q^e - \int_0^2 (12 + 2Q) \, dQ = 16*2 - 12Q - Q^2 = 32 - 12*2 - 2^2 = 4\]

7.6 Consumer surplus (CS) (Monopoly vs competition)

7.61 \[Q \text{ and } P \text{ if maximum profit as target}\]
- \[AR = 30 - 2Q\]
  \[TR = 30Q - 2Q^2\]
  \[MR = 30 - 4Q\]
- \[MC = MR\]
  \[12 = 30 - 4Q\]
  \[4Q = 18\]
  \[Q = 4.5\]
  \[P = 30 - 2*4.5 = 21\]
7.62 • Prices Monopoly: 21 Competition: 12
Quantities Monopoly: 4.5 Competition: 12 = 30 - 2Q = 9

• CS Monopoly
\[ CS_{Monopoly} = \int \left(30 - 2Q \right) dQ - P^e Q^e = 30Q - Q^2 - 21 \times 4.5 \]
\[ = 30 \times 4.5 - 4.5^2 - 94.5 = 135 - 20.25 - 94.5 = 20.25 \]

• CS Competition
\[ CS_{Competition} = \int \left(30 - 2Q \right) dQ - P^e Q^e = 30Q - Q^2 - 12 \times 9 \]
\[ = 30 \times 9 - 9^2 - 108 = 270 - 81 - 108 = 81 \]

7.63

7.7 CS, PS and the effects of a per unit tax

CS = 0.5*(240 - 168)*12 = 432
PS = 0.5*(168 - 120)*12 = 288
CS + PS = 432 + 288 = 720

CS = 0.5*(240 - 180)*10 = 300
PS = 0.5*(160 - 120)*10 = 200
Tax receipts = 10*20 = 200
Deadweight loss = 720-300-200-200 = 20

→ Back to questions. Click here!