Microeconomics and mathematics (with answers)
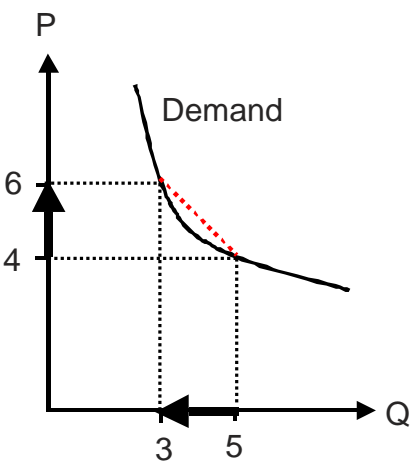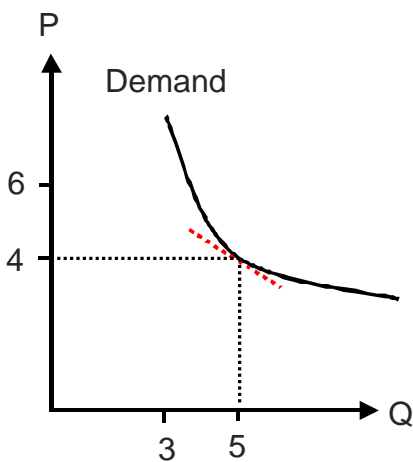## 3   Elasticities

I   Elasticities

Elasticity	Formula (arc elasticity)	Characteristics
Price elasticity of demand (e)	$e = \dfrac{\% \text{ change in quantity demanded}}{\% \text{ change in price}}$ $\rightarrow$ (e in absolute values)	• Perfectly elastic demand (e = infinite) • Perfectly inelastic demand (e = 0) • Unit elastic demand (e = 1) • Elastic demand (e > 1) • Inelastic demand (e < 1)
Cross-price elasticity of demand (Ce)	$Ce = \dfrac{\% \text{ change in quantity demanded good X}}{\% \text{ change in the price good Y}}$	• Substitutes (Ce > 0) • Complements (Ce < 0)

Income elasticity of demand (Ie)	$Ie = \dfrac{\% \text{ change in quantity demanded}}{\% \text{ change in income}}$	• Inferior goods (Ie < 0)
		 • Normal goods (Ie > 0); in addition:
		   •• Income elastic (Luxuries) (Ie > 1)
		   •• Income inelastic (Necessities) (Ie < 1)

Price elasticity of supply (Se)	$Se = \dfrac{\% \text{ change in quantity supplied}}{\% \text{ change in price}}$	

II   Arc elasticity vs point elasticity

Arc elasticity	Point elasticity
The variables concerned are observed in a certain range, for example changes in price from 4 to 6 which reduces Q from 5 to 3.	The variables concerned are observed at a certain point, for example infinitesimal changes in price at (5,4).
Price elasticity of demand (e): (P from 4 to 6, Q from 5 to 3) $= \dfrac{-40\,\%}{+50\,\%} = -0.8$ $\rightarrow$ **e = 0.8** **In this chapter, point elasticities are to be calculated.** ® ® ® ® ® ® ® ® ® ® ® ® ® ® ®	[We assume: $P = \dfrac{1}{Q}$; hence $Q = \dfrac{1}{P}$ and $\dfrac{dQ}{dP} = -1/P^2$] Price elasticity of demand at (5,4) = $\dfrac{dQ/Q}{dP/P} = \dfrac{dQ}{dP} * \dfrac{P}{Q} = -\dfrac{1}{16} * \dfrac{4}{5} = -\dfrac{1}{20} \rightarrow$ **e = $\dfrac{1}{20}$**

3.1	**Price elasticity of demand (e)**
	3.11 Calculate e when P = 4 Demand: Q = 80 - 2P
	3.12 Characterize the good ($\rightarrow$ Page 1, I Elasticities).
3.2	**Price elasticity of demand (e)**

	3.21 Calculate e when Q = 5
	          Demand: 2P = 30 - 2Q
	          (Hint: First transform demand into Q = ...)

	3.22 Characterize the good ($\rightarrow$ Page 1, I Elasticities).
3.3	**Price elasticity of demand (e)**
	Calculate e when P = 4 Demand: Q = 30 - 4P - 0.25P^2
3.4	**Point elasticities (formulae)**
	Analogous to the formula of the price elasticity of demand, draw up the formulae for the following elasticities:
	3.41 Price elasticity of supply (Se)
	3.42 Income elasticity of demand (Ie)
	3.43 Cross-price elasticity of demand (Ce)
3.5	**Price elasticity of supply (Se)**
	Calculate Se when P = 10 Supply: Q = - 3 + 0.5P
3.6	**Price elasticity of supply (Se)**
	Calculate Se when P = 50 Supply: Q = 30 + 4P - 0.01P^2
3.7	**Price elasticity of demand (e) and income elasticity of demand (Ie)**
	Demand: Q = 200 - P + 0.02I (P = 5; I[=income] = 1000)
	3.71 Calculate e
	3.72 Calculate Ie
	3.73 Characterize the good ($\rightarrow$ Page 1, I Elasticities).

3.8	**Cross-price elasticity of demand (Ce)** All variables refer to the supplier of good X, exception PY (= Price other goods) $Q = 200 - P + 0.02I + 0.04PY$ when $P = 5$, $I = 1000$, $PY = 100$ 3.81 Calculate Ce. 3.82 Characterize the good X in relation to good Y ($\rightarrow$ Page 1, I Elasticities).
3.9	**Price elasticity of demand (e), cross-price elasticity of demand (Ce) and income elasticity of demand (Ie)** All variables refer to the supplier of good X, except PY (= Price other goods). Demand: $Q = 200 - 5P - 2PY + 0.2I$ when $P = 10$, $PY = 12$, $I = 1000$ 3.91 Calculate e. 3.92 Calculate Ce. 3.93 Calculate Ie. 3.94 Characterize good X ($\rightarrow$ Page 1, I Elasticities).
3.10	**Price elasticity of demand (e), cross-price elasticity of demand (Ce) and income elasticity of demand (Ie)** All variables refer to the supplier of good X, except PY (= Price other goods). Demand: $Q = 180 - 5P + 10PY + 0.4I$ when $P = 6$, $PY = 8$, I(= income) $= 2000$ 3.101 Calculate e. 3.102 Calculate Ce. 3.103 Calculate Ie. 3.104 Characterize good X ($\rightarrow$ Page 1, I Elasticities).

$\rightarrow$ Answers. Click here!

Answers *Microeconomics* and mathematics

3 Elasticities

3.1	**Price elasticity of demand (e)**

3.11 e:

$$\frac{dQ}{dP} = -2 \qquad\qquad Q = 80 - 2*4 = 72$$

$$\frac{dQ}{dP} * \frac{P}{Q} = -2 * \frac{4}{72} = -\frac{8}{72} = -\frac{1}{9}$$

$$e = \frac{1}{9}$$

3.12 Demand is **inelastic** (e < 1).

3.2	**Price elasticity of demand (e)**

3.21 e:

2P = 30 - 2Q
2Q = 30 - 2P
Q = 15 - P

$$\frac{dQ}{dP} = -1 \qquad\qquad P: \quad Q = 15 - P \quad \rightarrow \quad 5 = 15 - P \quad \rightarrow \quad P = 10$$

$$\frac{dQ}{dP} * \frac{P}{Q} = -1 * \frac{10}{5} = -2$$

$$e = 2$$

3.22 Demand is **elastic** (e > 1).

3.3	**Price elasticity of demand (e)**

e:

$$\frac{dQ}{dP} = -4 - 0.5P = -4 - 0.5*4 = -6 \qquad\qquad Q: \quad Q = 30 - 16 - 4 = 10$$

$$\frac{dQ}{dP} * \frac{P}{Q} = -6 * \frac{4}{10} = -2.4$$

$$e = 2.4$$

3.4	**Point elasticities**

(where not stated otherwise, Q refers to the quantity demanded)

3.41 $Se = \dfrac{dQ}{dP} * \dfrac{P}{Q}$ (Q = quantity supplied)

3.42 $Ie = \dfrac{dQ}{dI} * \dfrac{I}{Q}$

3.43 $Ce = \dfrac{dQx}{dPY} * \dfrac{Py}{Qx}$ (x, y: goods)

3.5	**Price elasticity of supply (Se)** (Q refers to the quantity supplied) Se: $$\frac{dQ}{dP} = 0.5 \qquad\qquad Q = -3 + 0.5*10 = 2$$ $$\mathbf{Se} = \frac{dQ}{dP} * \frac{P}{Q} = 0.5 * \frac{10}{2} = \mathbf{2.5}$$
3.6	**Price elasticity of supply (Se)** (Q refers to the quantity supplied) Se: $$\frac{dQ}{dP} = 4 - 0.02P = 4 - 0.02*50 = 3$$ $$Q = 30 + 4*50 - 0.01*50^2 = 30 + 200 - 25 = 205$$ $$\mathbf{Se} = \frac{dQ}{dP} * \frac{P}{Q} = 3 * \frac{50}{205} = \mathbf{0.73}$$
3.7	**Price elasticity of demand (e) and income elasticity of demand (Ie)** 3.71 e: $$\frac{dQ}{dP} = -1 \qquad\qquad Q = 200 - 5 + 20 = 215$$ $$\frac{dQ}{dP} * \frac{P}{Q} = -1 * \frac{5}{215} = -0.02$$ $$\mathbf{e = 0.02}$$ 3.72 Ie: $$\frac{dQ}{dI} = 0.02$$ $$\mathbf{Ie} = \frac{dQ}{dI} * \frac{I}{Q} = 0.02 * \frac{1000}{215} = \mathbf{0.09}$$ 3.73 Demand is **inelastic** (e < 1); the good is a **normal good** (Ie > 0); the demand is **income-inelastic** (Ie < 1), hence the good is a **necessity**.
3.8	**Cross-price elasticity of demand (Ce)** 3.81 Ce: $$\frac{dQ}{dP_Y} = 0.04 \qquad\qquad Q = 200 - 5 + 0.02*1000 + 0.04 * 100 = 219$$ $$\mathbf{Ce} = \frac{dQ}{dP_Y} * \frac{P_Y}{Q} = 0.04 * \frac{100}{219} = \mathbf{0.02}$$ 3.82 The goods X and Y are **substitutes** (Ce > 0).
3.9	**Price elasticity of demand (e), cross-price elasticity of demand (Ce) and income elasticity of demand (Ie)** $$Q = 200 - 5*10 - 2*12 + 200 = 200 - 50 - 24 + 200 = 326$$

3.9 cont.	3.91	e: $$\frac{dQ}{dP} = -5 \qquad e = \frac{dQ}{dP} * \frac{P}{Q} = -5 * \frac{10}{326} = -0.15 \quad \rightarrow \quad \textbf{e = 0.15}$$
	3.92	Ce: $$\frac{dQ}{dPY} = -2$$ $$\textbf{Ce} = \frac{dQ}{dPY} * \frac{PY}{Q} = -2 * \frac{12}{326} \textbf{ = - 0.07}$$
	3.93	Ie: $$\frac{dQ}{dI} = 0.2$$ $$Ie = \frac{dQ}{dI} * \frac{I}{Q} = 0.2 * \frac{1000}{326} = \textbf{0.61}$$
	3.94	e = 0.15 → The demand is **inelastic** (e < 1). Ce = - 0.07 → X and Y are **complements** (Ce < 0). Ie = 0.61 → X is a **normal good** and it is **income-inelastic** (necessity) (0 < Ie < 1).
3.10		**Price elasticity of demand (e), cross-price elasticity of demand (Ce) and income elasticity of demand (Ie)** Q = 180 - 5*6 + 10*8 +0.4*2000 = 180 - 30 + 80 + 800 = 1030
	3.101	e: $$\frac{dQ}{dP} = -5 \qquad \frac{dQ}{dP} * \frac{P}{Q} = -5 * \frac{6}{1030} = -0.03$$ **e = 0.03**
	3.102	Ce: $$\frac{dQ}{dPY} = 10$$ $$\textbf{Ce} = \frac{dQ}{dPY} * \frac{PY}{Q} = 10 * \frac{8}{1030} \textbf{ = 0.08}$$
	3.103	Ie: $$\frac{dQ}{dI} = 0.4$$ $$Ie = \frac{dQ}{dI} * \frac{I}{Q} = 0.4 * \frac{2000}{1030} = \textbf{0.78}$$
	3.94	e = 0.03 → The demand is **inelastic** (e < 1). Ce = 0.08 → X and Y are **substitutes** (Ce > 0). Ie = 0.78 → X is a **normal good** and it is **income-inelastic** (necessity) (0 < Ie < 1).